

MPHYCC-6

M.Sc. Sem II

Plasma Physics

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Magnetosonic Waves

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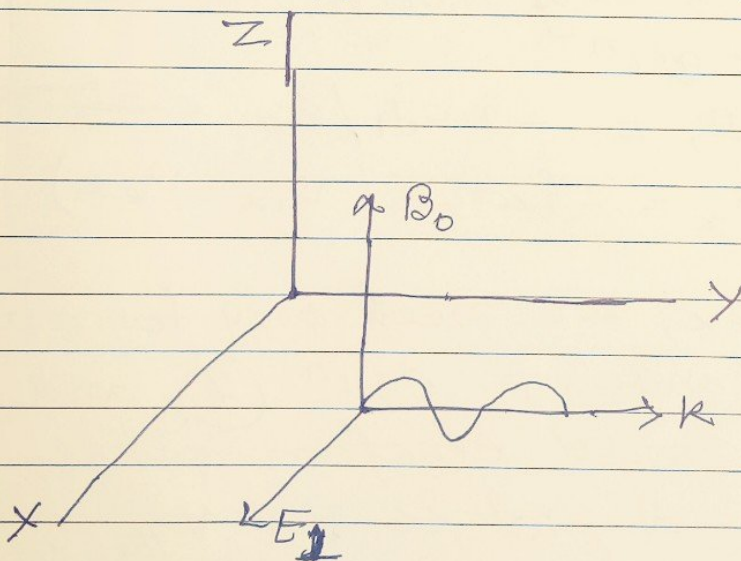
We consider low-frequency electro-magnetic waves propagating across B_0 . Again we may take $B_0 = B_0 \hat{z}$ and $E_1 = E_1 \hat{x}$, but we now let $K = K \hat{y}$ (fig). Now we see that the $E_1 \times B_0$ drifts lie along K , so that the plasma will be compressed and released in the course of the oscillation. It is necessary, therefore, to keep the ∇p term in the equation of motion. For the ions, we have

$$M n_0 \frac{\partial v_{i1}}{\partial t} = e n_0 (E_1 + v_{i1} \times B_0) - \gamma_i K T_i \nabla n_1 \quad \text{--- (1)}$$

With our choice of E_1 and K , this becomes

$$v_{ix} = \frac{ie}{m\omega} (E_x + v_{iy} B_0) \quad \text{--- (2)}$$

$$v_{iy} = \frac{ie}{m\omega} (-v_{ix} B_0) + \frac{k}{\omega} \frac{\gamma_i K T_i}{M} \frac{n_1}{n_0} \quad \text{--- (3)}$$



Figure

Geometry of a Magnetosonic Wave Propagating at right angles to B_0 .

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The equation of continuity yields

$$\frac{n_1}{n_0} = \frac{k}{\omega} v_{zy} \quad \text{--- (4)}$$

So that equⁿ (3) becomes

$$v_{zy} = -\frac{ie}{M\omega} v_{ix} B_0 + \frac{k^2}{\omega^2} \frac{\gamma_i k T_i}{M} v_{zy} \quad \text{--- (5)}$$

with the abbreviation

$$A = \frac{k^2}{\omega^2} \frac{\gamma_i k T_i}{M}$$

This becomes

$$v_{zy} (1-A) = -\frac{i\Omega_c}{\omega} v_{ix} \quad \text{--- (6)}$$

Combining this with equⁿ (2), we have

$$v_{ix} = \frac{ie}{M\omega} E_x + \frac{i\Omega_c}{\omega} \left(-\frac{i\Omega_c}{\omega} \right) (1-A)^{-1} v_{ix}$$

$$v_{ix} \left(1 - \frac{\Omega_c^2}{\omega^2} \right) = \frac{ie}{M\omega} E_x \quad \text{--- (7)}$$

This is the only component of v_{\perp} we shall need, since the only nontrivial component of the wave equⁿ

$$(\omega^2 - c^2 k^2) E_x = -i\omega \hat{j}_x / \epsilon_0 \quad \text{is}$$

$$\epsilon_0 (\omega^2 - c^2 k^2) E_x = -i\omega n_0 e (v_{ix} - v_{ex}) \quad \text{--- (8)}$$

To obtain v_{ex} , we need only to make the appropriate changes in equⁿ (7) and take the limit of small electron mass, so that $\omega^2 \ll \omega_p^2$ and $\omega^2 \ll k^2 v_{the}^2$.

$$V_{ex} = ie \frac{\omega^2}{m\omega} \frac{\omega^2}{\omega_c^2} \left(1 - \frac{k^2 Y_e K T_e}{\omega^2 m} \right) E_x \rightarrow$$

$$- \frac{i k^2 Y_e K T_e}{\omega B_0^2 e} E_x \quad \text{--- (9)}$$

Putting the last three equations together we have —

$$\epsilon_0 (\omega^2 - c^2 k^2) E_x = -i \omega n_0 e \left[\frac{ie}{m\omega} E_x \left(\frac{1-A}{1-A - (\Omega_c^2/\omega^2)} \right) \right. \\ \left. + \frac{i k^2 M Y_e K T_e}{\omega B_0^2 e M} E_x \right] \quad \text{--- (10)}$$

we shall again assume $\omega^2 \ll \Omega_c^2$, so that $1-A$ can be neglected relative to Ω_c^2/ω^2 , with the help of the definitions of Ω_p and v_A . we have

$$(\omega^2 - c^2 k^2) = - \frac{\Omega_p^2}{\Omega_c^2} \omega^2 (1-A) + \frac{k^2 c^2 Y_e K T_e}{v_A^2 M} \quad \text{--- (11)}$$

$$\omega^2 - c^2 k^2 \left(1 + \frac{Y_e K T_e}{M v_A^2} \right) + \frac{\Omega_p^2}{\Omega_c^2} \left(\omega^2 - k^2 \frac{Y_i K T_i}{M} \right) = 0$$

Since,

$$\frac{\Omega_p^2}{\Omega_c^2} = \frac{c^2}{v_A^2} \quad \text{--- (12)}$$

equation (11) becomes

$$\omega^2 \left(1 + \frac{c^2}{v_A^2} \right) = c^2 k^2 \left(1 + \frac{Y_e K T_e + Y_i K T_i}{M v_A^2} \right) =$$

$$c^2 k^2 \left(1 + \frac{v_s^2}{v_A^2} \right) \quad \text{--- (13)}$$

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where v_s is the acoustic speed. Finally, we have

$$\boxed{\frac{\omega^2}{k^2} = c^2 \frac{v_s^2 + v_A^2}{c^2 + v_A^2}}$$

(14)

This is the dispersion for the magnetosonic wave propagating perpendicular to B_0 . It is an acoustic wave in which the compressions and rarefactions are produced not by motions along E , but by $E \times B$ drifts across E .

In the limit $B_0 \rightarrow 0$, $v_A \rightarrow 0$, the magnetosonic wave turns into an ordinary ion acoustic wave. In the limit $kT \rightarrow 0$, $v_s \rightarrow 0$, the pressure gradient forces vanish, and the wave becomes a modified Alfvén wave. The phase velocity of the magnetosonic mode is almost always larger than v_A ; for this reason, it is often called simply the "fast" hydro magnetic wave.

